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One loop renormalizability of spontaneously broken gauge theory with a product of gauge groups on noncommutative spacetime: the $\mathrm{U}(1) \times \mathrm{U}(1)$ case

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Abstract: A generalization of the standard electroweak model to noncommutative spacetime would involve a product gauge group which is spontaneously broken. Gauge interactions in terms of physical gauge bosons are canonical with respect to massless gauge bosons as required by the exact gauge symmetry, but not so with respect to massive ones; and furthermore they are generally asymmetric in the two sets of gauge bosons. On noncommutative spacetime this already occurs for the simplest model of $\mathrm{U}(1) \times \mathrm{U}(1)$. We examine whether the above feature in gauge interactions can be perturbatively maintained in this model. We show by a complete one loop analysis that all ultraviolet divergences are removable with a few renormalization constants in a way consistent with the above structure.

Keywords: Renormalization Regularization and Renormalons, Spontaneous Symmetry Breaking, Gauge Symmetry, Beyond Standard Model.

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Contents

1. Introduction

The simplest noncommutative (NC) spacetime is the one in which coordinates $\hat{x}$ satisfy the algebra, $[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}$, where $\theta_{\mu\nu}$ is a real, antisymmetric, constant $n \times n$ matrix in n dimensions. A possible way to formulate field theory on this NC spacetime is through the Moyal-Weyl correspondence. One starts with the Weyl definition of function on NC spacetime by the Fourier transform,

$$\hat{f}(\hat{x}) = \frac{1}{(2\pi)^{n/2}} \int d^n k \; e^{ik_\mu \hat{x}^\mu} \tilde{f}(k) \,, \tag{1.1}$$

where $\tilde{f}(k)$ also defines a function $f(x)$ on the ordinary commutative spacetime. This relationship is shared by the algebras of functions on the two respectively spacetimes if the ordinary product of functions on commutative spacetime is replaced by the following star product,

$$(f_1 \star f_2)(x) = \left[\exp\left(\frac{i}{2} \theta^{\mu\nu} \partial^x_\mu \partial^y_\nu \right) f_1(x) f_2(y) \right]_{y=x} . \tag{1.2}$$

It is in this sense that one may study NC field theory by studying its counterpart on commutative spacetime where the ordinary product of functions is replaced by the starred one [1].

Field theories on NC spacetime have some salient features that are in contrast with ordinary theories and remain to be better understood; for example, the causality and unitarity problem [2] for time-space noncommutativity and the ultraviolet-infrared(UV-IR) mixing [3]. Furthermore, it would be natural to ask whether it is possible to generalize gauge interactions to NC spacetime. An important ingredient in establishing the viability of the generalizaion as a quantum theory is its renormalizability. This is a task that has to be fulfilled before one can build up any realistic models. It is the purpose of this work to continue the pursue in this direction, especially towards constructing realistic models for electroweak interactions. Our known results in this aspect are mainly based on explicit analyses and a general proof for (non)renormalizability of gauge theory on NC spacetime is still lacking [4]. This occurs due essentially to the highly nonlocal character of NC field theory. The renormalizability of the exact U(1) [5] and U(N) [6] gauge theories has been established to one loop order, and that of the real ϕ^4 theory [7] to two loops. The situation in spontaneously broken gauge theories is more subtle, considering the problems already met with spontaneously broken global symmetries [8]. The cases for the broken U(1) [9] and U(2) [10] theories have been examined, both with an affirmative answer. And it would be plausible to expect that the latter result also applies to the U(N) ($N > 2$) case.

In this work we extend the study of spontaneously broken gauge theories on NC spacetime to those with a product of groups. Our basic considerations are as follows. In a spontaneously broken gauge theory with a single group, the gauge couplings of unbroken and broken gauge interactions are the same; and the gauge boson masses are also fixed by the group structure. For example, for U(N) broken down to U($N-1$) by a scalar field in the fundamental representation, all $N-1$ pairs of charged gauge bosons (W) have the same mass which is related to that of the single neutral gauge boson (Z) by $m_W = m_Z/\sqrt{2}$. This is indeed some distance to our goal of constructing realistic electroweak models. It might be that for this purpose we have to consider the case with a product of groups so that we can have more space for tuning couplings and masses. There is a new feature in this case that does not appear for a single gauge group, namely the interactions among physical gauge bosons which are the mixtures of states originally associated to different group factors. Since only some combined part of symmetries is left unbroken, these interactions are usually not in a canonical form as dictated by a gauge symmetry but have diverse though related coefficients. It is not clear whether these relations can still be consistently maintained by renormalization at the quantum level on NC spacetime. Furthermore, there are not many choices for possible products of groups due to restrictions on generalized gauge invariance on NC spacetime in the approach using the Moyal-Weyl correspondence. First, only the U(N) group is closed under generalized gauge transformations [11]. This explains the mass relation mentioned above since there is no freedom even for the U(1) part of the group once the kinetic terms are canonically normalized. Actually there is no consistent way to separate the U(N) group into the group factors of SU(N) and U(1) since the latter are always mixed up by generalized gauge transformations. Thus we may restrict to the product of U(N) factors. Second, a given matter multiplet can have at most two nontrivial representations under two of the group factors [12]. This arises because it is not well-defined to transform under more than two group factors due to the noncommutativity of the star product. For the purpose of studying spontaneous symmetry breaking it would be general enough to consider the model of U(N_1) $\times$ U(N_2) with scalars in the (anti-)fundamental representations. As a first step of the efforts, we shall be less ambitious in this work and consider the simplest case of $\mathrm{U}(1)_{Y_1} \times \mathrm{U}(1)_{Y_2} \to \mathrm{U}(1)_Q$ which is nontrivial

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as compared to the one on commutative spacetime because of NC self-interactions. We shall see that only the gauge interactions corresponding to $\mathrm{U}(1)_Q$ are in a canonical form while those of the massive gauge boson are not, and that their mixing interactions are asymmetric though related by the ratio of couplings. We shall check whether all of this can be maintained at one loop level. With only one massive gauge boson we cannot discuss the mass relation and its renormalization. But with the positive results achieved thus far and in this work it is tempting to expect that the same answer would be applicable to the much more complicated case of $\mathrm{U}(N) \times \mathrm{U}(1)$ or even $\mathrm{U}(N_1) \times \mathrm{U}(N_2)$.

An alternative formalism of NC field theory [13, 14] is based on the Seiberg-Witten map [15] which relates NC and commutative gauge fields and is solved by a series expansion in θ. While it is more flexible to gauge groups and representations, it is not clear how to handle with increasingly higher dimension operators as one goes to higher orders in couplings and θ. We shall follow below the naive approach using the star product though we are aware of the potential jeopardy at higher orders caused by the UV-IR mixing.

The paper is organized as follows. In the next section we first write down the model and emphasize its difference to the commutative case, and then introduce the renormalization constants. We demonstrate its one loop renormalizability in section 3 by a complete analysis of all 1PI Green's functions which may be divergent by power counting. We conclude with the last section. We show in the appendices the Feynman rules and counterterms of the model and the Feynman diagrams for the 1PI four point Green's functions computed in the text.

2. The model

2.1 Classical lagrangian

We assume that there are two gauge fields $G_{i\mu}$ $(i = 1, 2)$ corresponding to the two groups $\mathrm{U}(1)_{Y_i}$ with respect to both of which the complex scalar field Φ is charged. The generalized, starred gauge transformations are

$$\begin{aligned} G_{i\mu} \to G'_{i\mu} &= U_i \star G_{i\mu} \star U_i^{-1} + i g_i^{-1} U_i \star \partial_\mu U_i^{-1} \,, \\ \Phi \to \Phi' &= U_1 \star \Phi \star U_2^{-1} \,, \end{aligned} \tag{2.1}$$

where $U_i = \exp[i g_i \eta_i(x)]_\star$, and g_i are gauge couplings. Note that the transformation rule for Φ is unique up to interchanging the roles of the two group factors. This arises because of the following observation [12]. Although the two symmetries are commutative as global and internal ones, they are not so as position-dependent ones due to noncommutativity of the star product of ordinary functions. It would be unclear how to do group multiplication if we assigned a transformation rule like, e.g., $\Phi \to \Phi' = U_1 \star U_2 \star \Phi$. The classical action invariant under the above transformations is constructed from the following lagrangian,

$$\begin{aligned} \mathcal{L}_{\text{class}} = &-\frac{1}{4} G_{1\mu\nu} G_1^{\ \mu\nu} - \frac{1}{4} G_{2\mu\nu} G_2^{\ \mu\nu} + \\ &+ (D_\mu \Phi)^\dagger D^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda \Phi^\dagger \Phi \Phi^\dagger \Phi \,, \end{aligned} \tag{2.2}$$

where we have suppressed the star notation for brevity, and

$$\begin{aligned} G_{i\mu\nu} &= \partial_\mu G_{i\nu} - \partial_\nu G_{i\mu} - ig_i[G_{i\mu}, G_{i\nu}]\,, \\ D_\mu \Phi &= \partial_\mu \Phi - ig_1 G_{1\mu}\Phi + ig_2 \Phi G_{2\mu}\,, \end{aligned} \tag{2.3}$$

with $[A, B] \equiv A \star B - B \star A$.

The spontaneous symmetry breaking is triggered by the non-vanishing scalar VEV, assuming $\mu^2, \lambda > 0$,

$$\Phi = \phi + \phi_0\,, \qquad \phi = \frac{(\sigma + i\pi)}{\sqrt{2}}\,, \qquad \phi_0 = \frac{v}{\sqrt{2}}\,, \tag{2.4}$$

with $v = \sqrt{\mu^2/\lambda}$. The σ field is the physical Higgs boson with mass $m_\sigma = \sqrt{2\lambda v^2}$ and the π field is the would-be Goldstone boson. The physical gauge bosons are the massless photon A corresponding to the unbroken $\mathrm{U}(1)_Q$ and the massive Z with mass $m_Z = gv$, where

$$\begin{aligned} \begin{pmatrix} Z \\ A \end{pmatrix} &= \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} G_1 \\ G_2 \end{pmatrix}, \\ g = \sqrt{g_1^2 + g_2^2}\,, \qquad & c = \frac{g_1}{g}\,, \qquad s = \frac{g_2}{g}\,. \end{aligned} \tag{2.5}$$

In terms of the above fields, $\mathcal{L}_{\text{class}}$ is expanded as a sum of the pure gauge terms and those involving the scalar fields. The first ones can be cast into the following form,

$$\begin{aligned} \mathcal{L}_{2G} &= -\frac{1}{4}Z_1 Z_1 + \frac{1}{2}m_Z^2 Z_\mu Z^\mu - \frac{1}{4}A_1 A_1\,, \\ \mathcal{L}_{3G} &= \frac{1}{2} ig \left[\left(c^2 - s^2\right) Z_1 Z_2 + cs(A_1 A_2 + A_1 Z_2 + Z_1 M)\right], \\ \mathcal{L}_{4G} &= \frac{1}{4} g^2 \left[\left(c^6 + s^6\right) Z_2 Z_2 + c^2 s^2 (A_2 A_2 + 2 Z_2 A_2 + MM) + 2cs\left(c^2 - s^2\right) Z_2 M\right], \end{aligned} \tag{2.6}$$

where we have freely used the property of the star product, $\int d^n x fg = \int d^n x gf$, to organize terms, and the following notations for brevity,

$$\begin{aligned} Z_1 &= \partial_\mu Z_\nu - \partial_\nu Z_\mu\,, \qquad & Z_2 &= [Z_\mu, Z_\nu]\,, \\ A_1 &= \partial_\mu A_\nu - \partial_\nu A_\mu\,, \qquad & A_2 &= [A_\mu, A_\nu], \\ M &= [Z_\mu, A_\nu] - [Z_\nu, A_\mu]\,. \end{aligned} \tag{2.7}$$

The terms involving scalar fields are

$$\begin{aligned} \mathcal{L}_{\phi G} &= -m_Z Z_\mu \partial^\mu \pi\,, \\ \mathcal{L}_{2\phi} &= \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2}(\partial_\mu \pi)^2\,, \\ \mathcal{L}_{3\phi} &= -\lambda v \sigma \left(\sigma^2 + \pi^2\right)\,, \\ \mathcal{L}_{4\phi} &= -\lambda \left(\frac{1}{4}\left(\sigma^4 + \pi^4\right) + \sigma^2 \pi^2 - \frac{1}{2}\sigma\pi\sigma\pi\right), \end{aligned}$$

$$
\begin{aligned}
\mathcal{L}_{G2\phi} &= +igcsA^{\mu}\left((\partial_{\mu}\sigma\sigma - \sigma\partial_{\mu}\sigma) + (\partial_{\mu}\pi\pi - \pi\partial_{\mu}\pi)\right) + \\
&\quad + \frac{1}{2}ig(c^2 - s^2)Z^{\mu}\left((\partial_{\mu}\sigma\sigma - \sigma\partial_{\mu}\sigma) + (\partial_{\mu}\pi\pi - \pi\partial_{\mu}\pi)\right) + \\
&\quad + \frac{1}{2}gZ^{\mu}\left(\pi\partial_{\mu}\sigma - \partial_{\mu}\pi\sigma + \partial_{\mu}\sigma\pi - \sigma\partial_{\mu}\pi\right), \\
\mathcal{L}_{\phi 2G} &= gm_Z\left(ics[Z_{\mu}, A^{\mu}]\pi + Z_{\mu}Z^{\mu}\sigma\right), \\
\mathcal{L}_{2G2\phi} &= g^2\left(\frac{1}{2}(c^4 + s^4)Z_{\mu}Z^{\mu}(\sigma^2 + \pi^2) + c^2s^2(Z_{\mu}\sigma Z^{\mu}\sigma + Z_{\mu}\pi Z^{\mu}\pi)\right) + \\
&\quad + g^2c^2s^2\left(A_{\mu}A^{\mu}(\sigma^2 + \pi^2) - (A_{\mu}\sigma A^{\mu}\sigma + A_{\mu}\pi A^{\mu}\pi)\right) + \\
&\quad + \frac{1}{2}ig^2(c^2 - s^2)Z_{\mu}Z^{\mu}[\pi, \sigma] + \\
&\quad + g^2cs(c^2 - s^2)\left(\frac{1}{2}(\sigma^2 + \pi^2)\{A_{\mu}, Z^{\mu}\} - (A_{\mu}\sigma Z^{\mu}\sigma + A_{\mu}\pi Z^{\mu}\pi)\right) + \\
&\quad + ig^2cs\left(\frac{1}{2}[\pi, \sigma]\{A_{\mu}, Z^{\mu}\} - (A_{\mu}\sigma Z^{\mu}\pi - A_{\mu}\pi Z^{\mu}\sigma)\right).
\end{aligned}
\tag{2.8}
$$

Let us make a few remarks on the above classical lagrangian. On commutative spacetime, it would degenerate trivially into the Abelian Higgs model plus a non-interacting pure and exact U(1) sector. On NC spacetime, however, because of the additional quadratic term in the U(1) field strength and the noncommutativity of interactions, the exact U(1) sector not only self-interacts but also communicates with the Abelian Higgs sector. This makes the theory much more involved and nontrivial. The self-interations of the photon are canonical as required by the exact U(1) symmetry, with a gauge coupling of gcs. This is not the case with the Z boson corresponding to the broken symmetry. And the mixed interactions are also asymmetric with respect to the two bosons. This arises essentially from their asymmetric couplings to the scalar field which in turn result in the mixing between them as shown in eq. (2.5). On the other hand, though asymmetric, all of these interactions are related by the only two available gauge couplings. It is thus interesting to check whether these relations can be consistent with the removal of UV divergences at higher orders and thus be possibly maintained in perturbation theory.

2.2 Gauge fixing and ghost terms

The procedure of gauge fixing may be generalized directly from commutative theory with the ordinary product replaced by the starred one,

$$
\begin{aligned}
\mathcal{L}_{\text{g.f.}} &= -\frac{1}{2\xi_1}f_1f_1 - \frac{1}{2\xi_2}f_2f_2, \\
f_1 &= \partial^{\mu}G_{1\mu} + ig_1\xi_1\left(\phi^{\dagger}\phi_0 - \phi_0^{\dagger}\phi\right), \\
f_2 &= \partial^{\mu}G_{2\mu} - ig_2\xi_2\left(\phi^{\dagger}\phi_0 - \phi_0^{\dagger}\phi\right).
\end{aligned}
\tag{2.9}
$$

Denoting the ghost fields as c_i and $\bar{c}_i$ $(i = 1, 2)$ and using the BRS transformations,

$$
\begin{aligned}
\mathsf{s}G_{i\mu} &= \partial_{\mu}c_i + ig_i[c_i, G_{i\mu}], \\
\mathsf{s}\phi &= ig_1c_1\Phi - ig_2\Phi c_2,
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{s}\phi^\dagger &= ig_2 c_2 \Phi^\dagger - ig_1 \Phi^\dagger c_1 \,, \\
\mathsf{s}c_i &= ig_i c_i c_i \,, \\
\mathsf{s}\bar{c}_i &= -\frac{1}{\xi_i} f_i \,,
\end{aligned}
\tag{2.10}
$$

the ghost terms are constructed as

$$
\mathcal{L}_{\text{ghost}} = -\bar{c}_1 \mathsf{s} f_1 - \bar{c}_2 \mathsf{s} f_2 \,, \tag{2.11}
$$

where

$$
\begin{aligned}
\mathsf{s}f_1 &= \partial^\mu \left(\partial_\mu c_1 + ig_1 [c_1, G_{1\mu}]\right) + g_1^2 \xi_1 \left(\Phi^\dagger c_1 \phi_0 + \phi_0^\dagger c_1 \Phi\right) - \\
&\quad - g_1 g_2 \xi_1 \left(c_2 \Phi^\dagger \phi_0 + \phi_0^\dagger \Phi c_2\right) , \\
\mathsf{s}f_2 &= \partial^\mu (\partial_\mu c_2 + ig_2 [c_2, G_{2\mu}]) + g_2^2 \xi_2 \left(c_2 \Phi^\dagger \phi_0 + \phi_0^\dagger \Phi c_2\right) - \\
&\quad - g_1 g_2 \xi_2 \left(\Phi^\dagger c_1 \phi_0 + \phi_0^\dagger c_1 \Phi\right) .
\end{aligned}
\tag{2.12}
$$

Then, $\mathsf{s}(\mathcal{L}_{\text{g.f.}} + \mathcal{L}_{\text{ghost}}) = 0$ due to $\mathsf{s}^2 f_i = 0$.

To avoid unwanted quadratic $A - Z$ mixing in $\mathcal{L}_{\text{g.f.}}$, we work below in the simplified version of $\xi_1 = \xi_2 = \xi$. Introducing the diagonalized ghosts corresponding to the gauge bosons A and Z,

$$
\begin{aligned}
\begin{pmatrix} c_Z \\ c_A \end{pmatrix} &= \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} , \\
\begin{pmatrix} \bar{c}_Z \\ \bar{c}_A \end{pmatrix} &= \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} \bar{c}_1 \\ \bar{c}_2 \end{pmatrix} ,
\end{aligned}
\tag{2.13}
$$

we obtain,

$$
\begin{aligned}
\mathcal{L}_{\text{g.f.}} &= -\frac{1}{2\xi} \left((\partial^\mu Z_\mu)^2 + (\partial^\mu A_\mu)^2\right) - m_Z \pi \partial^\mu Z_\mu - \frac{1}{2} \xi m_Z^2 \pi^2 \,, \\
\mathcal{L}_{\text{ghost}} &= \mathcal{L}_{c\bar{c}} + \mathcal{L}_{\phi c\bar{c}} + \mathcal{L}_{Gc\bar{c}} \,,
\end{aligned}
\tag{2.14}
$$

where

$$
\begin{aligned}
\mathcal{L}_{c\bar{c}} &= -\bar{c}_A \partial^2 c_A - \bar{c}_Z (\partial^2 + \xi m_Z^2) c_Z , \\
\mathcal{L}_{\phi c\bar{c}} &= -\frac{1}{2} \xi g^2 v \bar{c}_Z \left(\{c_Z, \sigma\} + \left(c^2 - s^2\right) i [c_Z, \pi] + 2csi [c_A, \pi]\right) , \\
\mathcal{L}_{Gc\bar{c}} &= +igcs \left(\partial^\mu \bar{c}_A \left([c_A, A_\mu] + [c_Z, Z_\mu]\right) + \partial^\mu \bar{c}_Z \left([c_Z, A_\mu] + [c_A, Z_\mu]\right)\right) + \\
&\quad + ig \left(c^2 - s^2\right) \partial^\mu \bar{c}_Z [c_Z, Z_\mu] .
\end{aligned}
\tag{2.15}
$$

Note that the $Z\pi$ mixing term in $\mathcal{L}_{\text{g.f.}}$ is cancelled by $\mathcal{L}_{\phi G}$ in eq. (2.8). The complete Feynman rules are collected in appendix A.

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2.3 Renormalization constants and counterterms

Now we introduce renormalization constants for the bare quantities. It turns out that in the gauge sector it is convenient to introduce renormalization constants for the original gauge fields. We have,

$$\begin{aligned}
(G_{i\mu})_{\rm B} &= Z_{G_i}^{1/2} G_{i\mu}, & (\phi)_{\rm B} &= Z_\phi^{1/2}\phi\,,\\
(g_i)_{\rm B} &= Z_{G_i}^{-1/2} Z_{g_i} g_i\,, & (\lambda)_{\rm B} &= Z_\phi^{-2} Z_\lambda \lambda\,,\\
(\mu^2)_{\rm B} &= Z_\phi^{-1}\mu^2\left(1+\frac{\delta\mu^2}{\mu^2}\right), & (v)_{\rm B} &= Z_\phi^{1/2} v\left(1+\frac{\delta v}{v}\right).
\end{aligned} \tag{2.16}$$

The redundant constant δv in the scalar sector will be determined by demanding vanishing σ tadpole at higher orders.

Since there is mixing between gauge bosons, there are two equivalent ways to proceed when separating counterterms from the bare lagrangian. One way is to start with eq. (2.16) and define the renormalized A and Z fields in terms of the renormalized c and s through eq. (2.5). This proves to be convenient for the gauge sector. The alternative way is to consider eq. (2.5) as a bare relation and introduce counterterms for the bare c and s which are in turn determined by δZ_{G_i} and δZ_{g_i}. This turns out to be better for organizing the counterterms in the gauge-scalar sector. In what follows, our c and s are always meant to be renormalized quantities when this differentiation is necessary.

For the gauge fixing and ghost part, the procedure is parallel to that in ref. [10] though slightly more complicated. We choose the quantities appearing in the gauge fixing functions f_i to be already renormalized, consider the BRS transformation of the renormalized fields, and then introduce the renormalization constants for the ghost fields,

$$(c_i)_{\rm B} = Z_{c_i} c_i\,. \tag{2.17}$$

The $c_{Z,A}$ fields are again given by eq. (2.13) in terms of the renormalized quantities.

We shall not present the lengthy expressions for the counterterms whose Feynamn rules are listed in appendix A. We just comment that there are counterterms to vertices which do not appear at tree level. This arises from the mixing of the A, Z fields and their different renormalization.

3. One loop renormalizability

In this section we present our one loop results on UV divergences and demonstrate explicitly that the model is renormalizable at one loop. We consider only diagrams which may be divergent by power counting, but exclude exceptional external momentum configurations such as $\theta_{\mu\nu}p^\nu = 0$ which may cause the UV-IR mixing. We work for simplicity in the $\xi_1 = \xi_2 = \xi = 1$ gauge and use the dimensional regularization in $n = 4 - 2\epsilon$ dimensions for the UV divergence.

We have exhausted all possible one- to four-point 1PI Green's functions, but it is unnecessary to list here the lengthy results. Instead, we classify in table 1 all possible three- and four-point functions for clarity. The tadpole and two-point functions are easy

$\sigma\sigma\sigma, \sigma\pi\pi$		$\pi\pi\pi, \pi\sigma\sigma$
$A\sigma\sigma, A\pi\pi$ $Z\sigma\sigma, Z\pi\pi, Z\sigma\pi$	$A\sigma\pi$	
$ZZ\sigma, ZA\sigma$	$ZA\sigma$	$ZZ\pi, AA\sigma, AA\pi$
AAA, ZZZ, ZZA	ZAA	
$\sigma c_Z\bar{c}_Z, \pi c_Z\bar{c}_Z, \pi c_A\bar{c}_Z$	$\sigma c_A\bar{c}_Z$	$\sigma c_Z\bar{c}_A, \sigma c_A\bar{c}_A, \pi c_Z\bar{c}_A, \pi c_A\bar{c}_A$
$Ac_A\bar{c}_A, Ac_Z\bar{c}_Z$ $Zc_Z\bar{c}_Z, Zc_A\bar{c}_Z, Zc_Z\bar{c}_A$	$Zc_A\bar{c}_A$ $Ac_A\bar{c}_Z, Ac_Z\bar{c}_A$	
$\sigma\sigma\sigma\sigma, \pi\pi\pi\pi, \sigma\sigma\pi\pi$		$\sigma\sigma\sigma\pi, \sigma\pi\pi\pi$
$AA\sigma\sigma, AA\pi\pi$ $ZZ\sigma\sigma, ZZ\pi\pi, ZZ\sigma\pi$ $AZ\sigma\sigma, AZ\pi\pi, AZ\sigma\pi$	$AA\sigma\pi$	
$AAAA, ZZZZ, AAZZ, AZZZ$	$AAAZ$	

Table 1: All possible three- and four-point 1PI Green's functions which may be divergent by power counting. Listed in the first, second and third columns are respectively the vertices with both tree and counterterm contributions, with counterterm but without tree contributions, and without either.

to compute and not listed there. The computation of up to three-point functions is similar to ref. [10] and we refer to that reference for details. We shall show our calculation of four-point functions by some typical examples. But before doing that, we would like to mention that there are many cross-checks which guarantee the correctness of our results. Generally, there are a large number of Green's functions that have to be made finite by adjusting the ten renormalization constants in eqs. (2.16) and (2.17). Due to the $A-Z$ mixing there are vertices that do not appear at tree level but have counterterms (as shown in the second column in the table 1). The cancellation of divergences in these vertices serves as a nontrivial consistency check of the result. For some vertices, different terms have different couplings and momentum-dependent trigonometric functions, and are thus renormalized differently. This also serves as a nontrivial check of the calculation. Finally, those listed in the last column must be finite by themselves if the model is renormalizable.

Let us start with the $\phi\phi\phi\phi$-type vertex. The general Feynman diagrams are shown and numbered in appendix B. The simplest ones are the vertices $\sigma\sigma\sigma\sigma$ and $\pi\pi\pi\pi$. We present our result for the vertex $\sigma(p_1)\sigma(p_2)\pi(p_3)\pi(p_4)$ which is richer in structure, where p_i are the incoming momenta of the particles. The UV divergences are found as follows.

$$
\begin{aligned}
(a)_{15} = ig^4\Delta_\epsilon \Bigg\{ c_{12}c_{34}\Bigg[&\frac{3}{2}\left(c^2-s^2\right)^2 + 2c^2s^2 + \frac{1}{4}\left(c^2-s^2\right)^4 + \\
&+ 4c^4s^4 + 2\left(c^2-s^2\right)^2 c^2s^2 + \frac{1}{4}\Bigg] - c_{13,24}\left[\left(c^2-s^2\right)^2 + 2c^2s^2\right]\Bigg\} \\
= ig^4\Delta_\epsilon \left(c^4+s^4\right)&\left(2c_{12}c_{34} - c_{13,24}\right), \qquad (3.1)
\end{aligned}
$$

with $c_{ij} = \cos(p_i \wedge p_j)$, $s_{ij} = \sin(p_i \wedge p_j)$, $c_{ij,kl} = \cos(p_i \wedge p_j + p_k \wedge p_l)$, $s_{ij,kl} = \sin(p_i \wedge p_j + p_k \wedge p_l), p \wedge q = 1/2\theta_{\mu\nu}p^\mu q^\nu$ and $\Delta_\epsilon = 1/(16\pi^2\epsilon)$. $(x)_n$ means that there are n diagrams

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contributing to the type-(x) diagram shown in appendix B, not counting permutations of external identical particles. Note that simplified structures as above are usually reached only upon summing up permutated diagrams.

$$\begin{aligned}
(b)_8 &= i\lambda g^2\Delta_\epsilon\left[\left(c^2-s^2\right)^2+4c^2s^2+1\right]\left[-2c_{12}c_{34}+2(c_{13,24}-c_{12}c_{34})\right] \\
&= i\lambda g^2\Delta_\epsilon 4(c_{13,24}-2c_{12}c_{34})\,, \\
(c)_{12} &= ig^4\Delta_\epsilon\Big\{-c_{12}c_{34}\left[\left(c^4+s^4\right)\left(1+\left(c^2-s^2\right)^2\right)+8c^4s^4+4c^2s^2(c^2-s^2)^2\right]+ \\
&\qquad +2(s_{31}s_{24}+s_{41}s_{23})\left[(c^2-s^2)^2+2c^2s^2\right]\Big\} \\
&= ig^4\Delta_\epsilon 2(c^4+s^4)(c_{13,24}-2c_{12}c_{34})\,,
\end{aligned} \tag{3.2}$$

where we have used the momentum conservation and the antisymmetry of $\theta_{\mu\nu}$ to obtain, $s_{31}s_{24}+s_{41}s_{23}=c_{13,24}-c_{12}c_{34}$.

$$\begin{aligned}
(d)_3 &= i\lambda^2\Delta_\epsilon 8(2c_{12}c_{34}-c_{13,24})\,, \\
(e)_5 &= ig^4\Delta_\epsilon\Big\{4c_{12}c_{34}\left[\left(c^4+s^4\right)^2+4c^4s^4+2c^2s^2\left(c^2-s^2\right)^2\right]+ \\
&\qquad +4(s_{13}s_{24}+s_{14}s_{23})\left[(c^2-s^2)^2+2c^2s^2\right]\Big\} \\
&= ig^4\Delta_\epsilon 4(c^4+s^4)(2c_{12}c_{34}-c_{13,24})\,.
\end{aligned} \tag{3.3}$$

The total UV divergence of the vertex is then,

$$iV^{\sigma\sigma\pi\pi}(p_1,p_2,p_3,p_4)=i\Delta_\epsilon\left[3g^4\left(c^4+s^4\right)-4g^2\lambda+8\lambda^2\right](2c_{12}c_{34}-c_{13,24})\,. \tag{3.4}$$

The $GG\phi\phi$-type vertices involve the most types of diagrams. We illustrate our calculation by the $A_\mu(p_1)A_\nu(p_2)\pi(p_3)\sigma(p_4)$ vertex which has no tree level contribution but does have a counterterm.

$$\begin{aligned}
(a)_2 &= +ig^4\Delta_\epsilon g_{\mu\nu}2\left(c^2-s^2\right)c^2s^2\ c_{12}s_{34}\,, \\
(b)_2 &= -ig^4\Delta_\epsilon g_{\mu\nu}\frac{3}{2}\left(c^2-s^2\right)c^2s^2\ c_{12}s_{34}\,,
\end{aligned} \tag{3.5}$$

where once again the same structure as the counterterm is achieved only upon summing over permutated diagrams. The type-(c) diagram turns out to be finite as the highest power of the loop momentum k actually disappears, e.g., $k^\alpha k^\beta k_\nu P_{\alpha\beta\mu}(k+p_3,-k-p_1-p_3,p_1)=O(k^3)$. There are no type-$(d)$ and -(j) diagrams at all in this case since the photon couplings are diagonal in scalar fields. The type-(e) diagram is generally complicated because it involves two GGG vertices. But in the current case we only have a Z-loop to compute.

$$\begin{aligned}
(e)_1 &= +ig^4\Delta_\epsilon g_{\mu\nu}9\left(c^2-s^2\right)c^2s^2\ c_{12}s_{34}, \\
(f)_2 &= -ig^4\Delta_\epsilon g_{\mu\nu}2\left(c^2-s^2\right)c^2s^2\ c_{12}s_{34}, \\
(g)_2 &= +ig^4\Delta_\epsilon g_{\mu\nu}\frac{3}{2}\left(c^2-s^2\right)c^2s^2\ c_{12}s_{34}, \\
(h)_4 &= -ig^4\Delta_\epsilon g_{\mu\nu}4\left(c^2-s^2\right)c^2s^2\ c_{12}s_{34}, \\
(i)_4 &= +ig^4\Delta_\epsilon g_{\mu\nu}3\left(c^2-s^2\right)c^2s^2\ c_{12}s_{34}, \\
(k)_1 &= -ig^4\Delta_\epsilon g_{\mu\nu}6\left(c^2-s^2\right)c^2s^2\ c_{12}s_{34}, \\
(l)_2 &= +ig^4\Delta_\epsilon g_{\mu\nu}6\left(c^2-s^2\right)c^2s^2\ c_{12}s_{34}\,.
\end{aligned} \tag{3.6}$$

The overall divergence of the vertex is then,

$$iV^{AA\pi\sigma}_{\mu\nu}(p_1,p_2,p_3,p_4) = +ig^4\Delta_\epsilon g_{\mu\nu}8\left(c^2-s^2\right)c^2s^2\ c_{12}s_{34}\,. \tag{3.7}$$

The $GGGG$ vertices are the most difficult part of the computation due mainly to the complicated momentum dependent trigonometric structures. We choose as a typical example the $Z_\mu(p_1)Z_\nu(p_2)A_\alpha(p_3)A_\beta(p_4)$ vertex to show our calculation, which has a rich structure. Diagrams of type-(a), (c), (d) and (f) are easy to compute with the results:

$$\begin{aligned}
(a)_7 &= +ig^4\Delta_\epsilon c^2s^2\left[\left(c^2-s^2\right)^2+1\right]\times\\
&\quad\times\frac{2}{3}[g_{\mu\nu}g_{\alpha\beta}+g_{\mu\alpha}g_{\beta\nu}+g_{\mu\beta}g_{\nu\alpha}]\left[2c_{12}c_{34}+c_{13,24}\right],\\
(c)_5 &= -ig^4\Delta_\epsilon c^2s^2\left[\left(c^2-s^2\right)^2+2c^2s^2\right]\times\\
&\quad\times\frac{1}{6}[g_{\mu\nu}g_{\alpha\beta}+g_{\mu\alpha}g_{\beta\nu}+g_{\mu\beta}g_{\nu\alpha}]\left[2c_{12}c_{34}+c_{13,24}\right],\\
(d)_{10} &= -ig^4\Delta_\epsilon c^2s^2\left[\left(c^2-s^2\right)^2+1\right]\times\\
&\quad\times 4[c_{12}c_{34}g_{\mu\nu}g_{\alpha\beta}+c_{13}c_{24}g_{\mu\alpha}g_{\beta\nu}+c_{14}c_{23}g_{\mu\beta}g_{\nu\alpha}],\\
(f)_5 &= +ig^4\Delta_\epsilon c^2s^2 2\Big\{2(c^4+s^4)c_{12}c_{34}g_{\mu\nu}g_{\alpha\beta}+\\
&\qquad\qquad+\left[\left(c^2-s^2\right)^2+1\right]\left(c_{13}c_{24}g_{\mu\alpha}g_{\beta\nu}+c_{14}c_{23}g_{\mu\beta}g_{\nu\alpha}\right)\Big\}.
\end{aligned} \tag{3.8}$$

Diagrams (b), (e) and (g) involve multiple pure gauge vertices and are more complicated. For example, (b) contains a product of four triple-vertex $P_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3)$ tensors, whose highest power term in loop momentum k provides the UV divergence. Upon dropping oscillatory phases involving k and doing symmetric loop integration, we may use,

$$\begin{aligned}
&P_{\rho\sigma\alpha}(k,-k,0)P^{\sigma}_{\tau\beta}(k,-k,0)P^{\tau}_{\eta\mu}(k,-k,0)P^{\eta\rho}_{\nu}(k,-k,0)\to\\
&\qquad\qquad\to[47(g_{\mu\nu}g_{\alpha\beta}+g_{\mu\beta}g_{\nu\alpha})+17g_{\mu\alpha}g_{\beta\nu}]\frac{\left(k^2\right)^2}{12}\,.
\end{aligned}$$

Combining coefficients of c and s, the results are

$$\begin{aligned}
(b)_5 &= +ig^4\Delta_\epsilon c^2s^2\left(c^4+s^4\right)\times\\
&\quad\times\frac{1}{6}\Big\{c_{12,34}[47(g_{\mu\nu}g_{\alpha\beta}+g_{\mu\beta}g_{\nu\alpha})+17g_{\mu\alpha}g_{\beta\nu}]+\\
&\qquad+c_{12,43}[47(g_{\mu\nu}g_{\alpha\beta}+g_{\mu\alpha}g_{\beta\nu})+17g_{\mu\beta}g_{\nu\alpha}]+\\
&\qquad+c_{13,24}[47(g_{\mu\alpha}g_{\beta\nu}+g_{\mu\beta}g_{\nu\alpha})+17g_{\mu\nu}g_{\alpha\beta}]\Big\},\\
(e)_8 &= -ig^4\Delta_\epsilon c^2s^2\left(c^4+s^4\right)\times\\
&\quad\times 2\Big\{c_{12}c_{34}(13g_{\mu\nu}g_{\alpha\beta}+g_{\mu\beta}g_{\nu\alpha}+g_{\mu\alpha}g_{\beta\nu})+9s_{12}s_{34}(g_{\mu\alpha}g_{\beta\nu}-g_{\mu\beta}g_{\nu\alpha})+\\
&\qquad+c_{13}c_{24}(13g_{\mu\alpha}g_{\beta\nu}+g_{\mu\nu}g_{\alpha\beta}+g_{\mu\beta}g_{\nu\alpha})+9s_{13}s_{24}(g_{\mu\nu}g_{\alpha\beta}-g_{\mu\beta}g_{\nu\alpha})\\
&\qquad+c_{14}c_{23}(13g_{\mu\beta}g_{\nu\alpha}+g_{\mu\alpha}g_{\beta\nu}+g_{\mu\nu}g_{\alpha\beta})+9s_{14}s_{23}(g_{\mu\nu}g_{\alpha\beta}-g_{\mu\alpha}g_{\beta\nu})\Big\},
\end{aligned}$$

$$
\begin{aligned}
(g)_4 =& +ig^4\Delta_\epsilon c^2 s^2\left(c^4+s^4\right)\times \\
&\times 2\Big\{c_{12}c_{34}(4g_{\mu\nu}g_{\alpha\beta}+g_{\mu\beta}g_{\nu\alpha}+g_{\mu\alpha}g_{\beta\nu})+9s_{12}s_{34}(g_{\mu\alpha}g_{\beta\nu}-g_{\mu\beta}g_{\nu\alpha})+ \\
&\quad +2c_{12}c_{34}(5g_{\mu\nu}g_{\alpha\beta}-g_{\mu\alpha}g_{\beta\nu}-g_{\mu\beta}g_{\nu\alpha})+6s_{12}s_{34}(g_{\mu\alpha}g_{\beta\nu}-g_{\mu\beta}g_{\nu\alpha}) \\
&\quad +c_{13,24}(7g_{\mu\alpha}g_{\beta\nu}+7g_{\mu\beta}g_{\nu\alpha}-8g_{\mu\nu}g_{\alpha\beta})\Big\}. \qquad (3.9)
\end{aligned}
$$

Using momentum conservation and antisymmetry of $\theta_{\mu\nu}$ to deduce the following relations,

$$
\begin{aligned}
c_{ik}c_{jl} &= \frac{1}{2}[c_{ij}c_{kl}+s_{ij}s_{kl}+c_{ik,jl}]\,, \\
s_{ik}s_{jl} &= \frac{1}{2}[c_{ij}c_{kl}+s_{ij}s_{kl}-c_{ik,jl}]\,, \\
c_{ik,jl} &= c_{il,jk}\,, \qquad (3.10)
\end{aligned}
$$

where $\{i,j,k,l\}$ is a permutation of $\{1,2,3,4\}$, and expressing all trigonometric functions in terms of $c_{12}c_{34}, s_{12}s_{34}$ and $c_{13,24}$, we obtain the total UV divergence for the vertex,

$$
\begin{aligned}
iV^{ZZAA}_{\mu\nu\alpha\beta}(p_1,p_2,p_3,p_4) =& \; ig^4\Delta_\epsilon 2c^2s^2\left(c^4+s^4\right)\times \\
&\times\Big\{[c_{13,24}-c_{12}c_{34}]\,[2g_{\mu\nu}g_{\alpha\beta}-g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\nu}g_{\beta\mu}]- \\
&\quad -3s_{12}s_{34}\,[g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\nu}g_{\beta\mu}]\Big\}. \qquad (3.11)
\end{aligned}
$$

Our explicit one loop result may be summarized by the following set of renormalization constants in the MS scheme,

$$
\begin{aligned}
\delta Z_{G_1} &= \Delta_\epsilon 3g^2c^2\,, & \delta Z_{G_2} &= \Delta_\epsilon 3g^2s^2\,, \\
\delta Z_{c_1} &= \Delta_\epsilon g^2c^2\,, & \delta Z_{c_2} &= \Delta_\epsilon g^2s^2\,, \\
\delta Z_{\phi} &= \Delta_\epsilon 2g^2\,, & & \\
\delta Z_{g_1} &= -\Delta_\epsilon 2g^2c^2\,, & \delta Z_{g_2} &= -\Delta_\epsilon 2g^2s^2\,, \\
\lambda\delta Z_\lambda &= \frac{\Delta_\epsilon}{2}\left[3g^4\left(c^4+s^4\right)-4g^2\lambda+8\lambda^2\right], & & \\
\frac{\lambda\delta\mu^2}{\mu^2} &= -\frac{\Delta_\epsilon}{2}\left[3g^4(1+2c^2s^2)+2g^2\lambda+4\lambda^2\right], & \frac{\delta v}{v} &= \Delta_\epsilon g^2\,. \qquad (3.12)
\end{aligned}
$$

The complete calculation shows that the above set is sufficient to remove all UV divergences that appear at one loop order. We emphasize this is true in the gauge sector though gauge interactions are not symmetric with respect to the two physical gauge bosons. The renormalization constants for the diagonalized fields and their mixings are,

$$
\begin{aligned}
\delta Z_A &= \Delta_\epsilon 6g^2c^2s^2, & \delta Z_Z &= \Delta_\epsilon 3g^2\left(c^4+s^4\right), \\
\delta Z_{c_A} &= \Delta_\epsilon 2g^2c^2s^2, & \delta Z_{c_Z} &= \Delta_\epsilon g^2\left(c^4+s^4\right), \\
\delta Z_{AZ} &= \Delta_\epsilon 3g^2cs\left(c^2-s^2\right), & \delta Z_{c_Ac_Z} &= \Delta_\epsilon g^2cs\left(c^2-s^2\right). \qquad (3.13)
\end{aligned}
$$

And the counterterm for the gauge boson mass and the renormalization constant for the exact U(1) coupling $e=gcs$ defined by $(e)_{\mathrm{B}}=Z_A^{-1/2}Z_e e$ are

$$
\delta m_Z^2 = m_Z^2\Delta_\epsilon g^2\left[4-\left(c^4+s^4\right)\right], \qquad \delta Z_e = -\Delta_\epsilon 4e^2\,. \qquad (3.14)
$$

4. Conclusion

A generalization of the standard electroweak model to NC spacetime would involve a product gauge group which is spontaneously broken. A criterion to consider this as a viable quantum field theory should include its perturbative renormalizability. We pointed out that there are two features in such a model which do not appear in the case of a single gauge group. Firstly, the gauge boson mass relation is determined jointly by the group structure and the ratio of gauge couplings. This may allow for more space for tuning the masses as happens in the standard model. Secondly, the gauge interactions of massless gauge bosons are canonical as required by exact gauge symmetry, but those of massive ones are generally not. The mixed interactions between the two sets of gauge bosons are also asymmetric though related, even if we start with a symmetric arrangement of group factors like $\mathrm{U}(N) \times \mathrm{U}(N)$. It is the purpose of the current work to examine whether these features can be consistently maintained at higher orders in perturbation theory so that such a model may still be renormalizable on NC spacetime. Due to technical complications, we have restricted to the simplest case of $\mathrm{U}(1)_{Y_1} \times \mathrm{U}(1)_{Y_2} \to \mathrm{U}(1)_Q$ as a first step in these efforts. Although the first feature mentioned above never appears, the second one can be thoroughly explored. We found indeed all UV divergences at one loop level can be removed altogether with a few renormalization constants. Based on this result and those already achieved so far, it would be very natural to expect that the same conclusion also applies to the more general case with the $\mathrm{U}(N_1) \times \mathrm{U}(N_2)$ gauge group. Furthermore, while this result is far away from demonstrating renormalizability to all orders, it does lend support to the viewpoint that it is worthwhile to consider seriously building up realistic models of gauge interactions on NC spacetime though this seems to be rather difficult.

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A. Feynman rules

We list below the complete Feynman rules for the model with the gauge choice of $\xi_1 = \xi_2 = \xi$. All momenta are incoming and shown in the parentheses of the corresponding particles.

Propagators (momentum p):

$$A_\mu \;\sim\!\sim\!\sim\!\sim\!\sim\; A_\nu \quad = \frac{-i}{p^2}\left[g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right]$$

$$Z_\mu \;\sim\!\sim\!\sim\!\sim\!\sim\; Z_\nu \quad = \frac{-i}{p^2 - m_Z^2}\left[g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2 - \xi m_Z^2}\right]$$

$$\sigma \;-\,-\,-\,-\,-\; \sigma \quad = \frac{i}{p^2 - m_\sigma^2}$$

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$$\pi \;\text{- - - - - - - -}\; \pi \quad = \frac{i}{p^2 - \xi m_Z^2}$$

$$c_A \;\cdots\!\blacktriangleright\cdots\cdots\blacktriangleleft\!\cdots\; \bar{c}_A \quad = \frac{i}{p^2}$$

$$c_Z \;\cdots\!\blacktriangleright\cdots\cdots\blacktriangleleft\!\cdots\; \bar{c}_Z \quad = \frac{i}{p^2 - \xi m_Z^2}$$

$G\phi\phi$ vertices:

$$\begin{aligned}
A_\mu \sigma(p_1)\sigma(p_2) &= A_\mu \pi(p_1)\pi(p_2) \\
&= 2gcs(p_1 - p_2)_\mu \; s_{12} \\
Z_\mu \sigma(p_1)\sigma(p_2) &= Z_\mu \pi(p_1)\pi(p_2) \\
&= g\left(c^2 - s^2\right)(p_1 - p_2)_\mu \; s_{12} \\
Z_\mu \sigma(p_1)\pi(p_2) &= g(p_1 - p_2)_\mu \; c_{12}
\end{aligned} \tag{A.1}$$

where $s_{ij} = \sin(p_i \wedge p_j)$ and $c_{ij} = \cos(p_i \wedge p_j)$.

$GG\phi$ vertices:

$$\begin{aligned}
Z_\mu(p_1)Z_\nu(p_2)\sigma &= i2gm_Z g_{\mu\nu} \; c_{12} \\
Z_\mu(p_1)A_\nu(p_2)\pi &= i2gcsm_Z g_{\mu\nu} \; s_{12}
\end{aligned} \tag{A.2}$$

$GG\phi\phi$ vertices:

$$\begin{aligned}
Z_\mu(p_1)Z_\nu(p_2)\sigma(p_3)\sigma(p_4) &= Z_\mu(p_1)Z_\nu(p_2)\pi(p_3)\pi(p_4) \\
&= i2g^2 g_{\mu\nu}[\left(c^4 + s^4\right) \; c_{12}c_{34} + 2c^2 s^2 \; c_{13,24}] \\
Z_\mu(p_1)Z_\nu(p_2)\pi(p_3)\sigma(p_4) &= i2g^2\left(c^2 - s^2\right) g_{\mu\nu} \; c_{12}s_{34} \\
A_\mu(p_1)A_\nu(p_2)\sigma(p_3)\sigma(p_4) &= A_\mu(p_1)A_\nu(p_2)\pi(p_3)\pi(p_4) \\
&= i4g^2 c^2 s^2 g_{\mu\nu}[c_{12}c_{34} - c_{13,24}] \\
A_\mu(p_1)Z_\nu(p_2)\sigma(p_3)\sigma(p_4) &= A_\mu(p_1)Z_\nu(p_2)\pi(p_3)\pi(p_4) \\
&= i2g^2 cs\left(c^2 - s^2\right) g_{\mu\nu}[c_{12}c_{34} - c_{13,24}] \\
A_\mu(p_1)Z_\nu(p_2)\pi(p_3)\sigma(p_4) &= i2g^2 cs g_{\mu\nu}[c_{12}s_{34} + s_{13,24}]
\end{aligned} \tag{A.3}$$

where $s_{ij,kl} = \sin(p_i \wedge p_j + p_k \wedge p_l)$ and $c_{ij,kl} = \cos(p_i \wedge p_j + p_k \wedge p_l)$.

GGG vertices:

$$\begin{aligned}
A_\alpha(p_1)A_\beta(p_2)A_\gamma(p_3) &= -2gcs \; s_{12} \; P_{\alpha\beta\gamma}(p_1, p_2, p_3) \\
Z_\alpha(p_1)Z_\beta(p_2)Z_\gamma(p_3) &= -2g\left(c^2 - s^2\right) \; s_{12} \; P_{\alpha\beta\gamma}(p_1, p_2, p_3) \\
Z_\alpha(p_1)Z_\beta(p_2)A_\gamma(p_3) &= -2gcs \; s_{12} \; P_{\alpha\beta\gamma}(p_1, p_2, p_3)
\end{aligned} \tag{A.4}$$

where

$$P_{\alpha\beta\gamma}(p_1, p_2, p_3) = (p_1 - p_2)_\gamma g_{\alpha\beta} + (p_2 - p_3)_\alpha g_{\beta\gamma} + (p_3 - p_1)_\beta g_{\gamma\alpha} \,.$$

Some simple properties of it are useful:

$$\begin{aligned}
P_{\alpha\beta\gamma}(p_1, p_2, p_3) &= -P_{\alpha\gamma\beta}(p_1, p_3, p_2) \\
&= -P_{\beta\alpha\gamma}(p_2, p_1, p_3) = -P_{\gamma\beta\alpha}(p_3, p_2, p_1)\,, \\
P_{\alpha\beta\gamma}(p_1, p_2, p_3) + P_{\gamma\alpha\beta}(p_1, p_2, p_3) &= P_{\beta\alpha\gamma}(p_1, p_3, p_2)\,.
\end{aligned} \tag{A.5}$$

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$GGGG$ vertices:

$$
\begin{aligned}
& A_{\mu_1}(p_1)A_{\mu_2}(p_2)A_{\mu_3}(p_3)A_{\mu_4}(p_4) = \\
& \quad = -i4g^2c^2s^2\left[g^A_{\mu_1\mu_2,\mu_3\mu_4}\ s_{12}s_{34} + g^A_{\mu_3\mu_1,\mu_2\mu_4}\ s_{31}s_{24} + g^A_{\mu_2\mu_3,\mu_1\mu_4}\ s_{23}s_{14}\right] \\
& Z_{\mu_1}(p_1)Z_{\mu_2}(p_2)Z_{\mu_3}(p_3)Z_{\mu_4}(p_4) = \\
& \quad = -i4g^2\left(c^6+s^6\right)\left[g^A_{\mu_1\mu_2,\mu_3\mu_4}\ s_{12}s_{34} + g^A_{\mu_3\mu_1,\mu_2\mu_4}\ s_{31}s_{24} + g^A_{\mu_2\mu_3,\mu_1\mu_4}\ s_{23}s_{14}\right] \\
& Z_{\mu_1}(p_1)Z_{\mu_2}(p_2)A_{\mu_3}(p_3)A_{\mu_4}(p_4) = \\
& \quad = i2g^2c^2s^2\left[g^S_{\mu_1\mu_2,\mu_3\mu_4}(c_{13,24}-c_{12}c_{34}) - 3g^A_{\mu_1\mu_2,\mu_3\mu_4}\ s_{12}s_{34}\right] \\
& Z_{\mu_1}(p_1)Z_{\mu_2}(p_2)Z_{\mu_3}(p_3)A_{\mu_4}(p_4) = \\
& \quad = i4g^2cs\left(c^2-s^2\right)\Big[g_{\mu_4\mu_1}g_{\mu_2\mu_3}(c_{43,12}-c_{41}c_{23}) + \\
& \qquad\qquad + g_{\mu_4\mu_2}g_{\mu_3\mu_1}(c_{41,23}-c_{42}c_{31}) + g_{\mu_4\mu_3}g_{\mu_1\mu_2}(c_{42,31}-c_{43}c_{12})\Big]
\end{aligned} \tag{A.6}
$$

where

$$
\begin{aligned}
g^A_{\mu_1\mu_2,\mu_3\mu_4} &= g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_4}g_{\mu_2\mu_3}\,, \\
g^S_{\mu_1\mu_2,\mu_3\mu_4} &= 2g_{\mu_1\mu_2}g_{\mu_3\mu_4} - g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_4}g_{\mu_2\mu_3}\,.
\end{aligned} \tag{A.7}
$$

$\phi\phi\phi$ vertices:

$$
\begin{aligned}
\sigma\sigma(p_1)\sigma(p_2) &= -i6\lambda v\ c_{12} \\
\sigma\pi(p_1)\pi(p_2) &= -i2\lambda v\ c_{12}
\end{aligned} \tag{A.8}
$$

$\phi\phi\phi\phi$ vertices:

$$
\begin{aligned}
\sigma(p_1)\sigma(p_2)\sigma(p_3)\sigma(p_4) &= \pi(p_1)\pi(p_2)\pi(p_3)\pi(p_4) \\
&= -i2\lambda[c_{12}c_{34}+c_{31}c_{24}+c_{23}c_{14}] \\
\sigma(p_1)\sigma(p_2)\pi(p_3)\pi(p_4) &= -i2\lambda[2c_{12}c_{34}-c_{13,24}]
\end{aligned} \tag{A.9}
$$

$\phi c\bar{c}$ vertices:

$$
\begin{aligned}
\sigma c_Z(p_1)\bar{c}_Z(p_2) &= -i\xi g^2 v\ c_{21} \\
\pi c_Z(p_1)\bar{c}_Z(p_2) &= -i\xi g^2 v\left(c^2-s^2\right)s_{21} \\
\pi c_A(p_1)\bar{c}_Z(p_2) &= -i\xi g^2 v\ 2cs\ s_{21}
\end{aligned} \tag{A.10}
$$

$Gc\bar{c}$ vertices:

$$
\begin{aligned}
A_\mu c_A(p_1)\bar{c}_A(p_2) &= A_\mu c_Z(p_1)\bar{c}_Z(p_2) \\
&= 2gcs\ p_{2\mu}\ s_{21} \\
Z_\mu c_Z(p_1)\bar{c}_Z(p_2) &= 2g\left(c^2-s^2\right)p_{2\mu}\ s_{21} \\
Z_\mu c_A(p_1)\bar{c}_Z(p_2) &= Z_\mu c_Z(p_1)\bar{c}_A(p_2) \\
&= 2gcs\ p_{2\mu}\ s_{21}
\end{aligned} \tag{A.11}
$$

Counterterms for self-energies and mixings are listed below. Note the momentum p is the incoming momentum of the gauge boson in the $G\phi$ mixing.

$$\sigma \; - - - - - - - \times = i\lambda v^3 \left[\frac{\delta\mu^2}{\mu^2} - \delta Z_\lambda - \frac{2\delta v}{v}\right]$$

$$\sigma \; - - - \times - - - \; \sigma = ip^2\delta Z_\phi - \frac{im_\sigma^2}{2}\left[\frac{-\delta\mu^2}{\mu^2} + 3\delta Z_\lambda + \frac{6\delta v}{v}\right]$$

$$\pi \; - - - \times - - - \; \pi = ip^2\delta Z_\phi - \frac{im_\sigma^2}{2}\left[\frac{-\delta\mu^2}{\mu^2} + \delta Z_\lambda + \frac{2\delta v}{v}\right]$$

$$A_\mu \sim\sim\times\sim\sim A_\nu = i(p_\mu p_\nu - p^2 g_{\mu\nu})(s^2\delta Z_{G_1} + c^2\delta Z_{G_2})$$

$$\begin{aligned} Z_\mu \sim\sim\times\sim\sim Z_\nu &= i(p_\mu p_\nu - p^2 g_{\mu\nu})(c^2\delta Z_{G_1} + s^2\delta Z_{G_2}) + \\ &\quad + ig_{\mu\nu}m_Z^2\left[2(c^2\delta Z_{g_1} + s^2\delta Z_{g_2}) + \frac{2\delta v}{v} + \delta Z_\phi\right] \end{aligned}$$

$$\begin{aligned} Z_\mu \sim\sim\times\sim\sim A_\nu &= i(p_\mu p_\nu - p^2 g_{\mu\nu})cs(\delta Z_{G_1} - \delta Z_{G_2}) + \\ &\quad + ig_{\mu\nu}m_Z^2 cs(\delta Z_{g_1} - \delta Z_{g_2}) \end{aligned}$$

$$Z_\mu \sim\sim\times - - - \; \pi = m_Z p_\mu\left[(c^2\delta Z_{g_1} + s^2\delta Z_{g_2}) + \frac{\delta v}{v} + \delta Z_\phi\right]$$

$$A_\mu \sim\sim\times - - - \; \pi = m_Z p_\mu cs(\delta Z_{g_1} - \delta Z_{g_2})$$

$$c_A \cdots\to\cdots\times\cdots\leftarrow\cdots \bar{c}_A = ip^2(s^2\delta Z_{c_1} + c^2\delta Z_{c_2})$$

$$\begin{aligned} c_Z \cdots\to\cdots\times\cdots\leftarrow\cdots \bar{c}_Z &= ip^2(c^2\delta Z_{c_1} + s^2\delta Z_{c_2}) - \\ &\quad - i\xi m_Z^2\left[(c^2\delta Z_{c_1} + s^2\delta Z_{c_2}) + (c^2\delta Z_{g_1} + s^2\delta Z_{g_2}) + \frac{\delta v}{v}\right] \end{aligned}$$

$$c_Z \cdots\to\cdots\times\cdots\leftarrow\cdots \bar{c}_A = ip^2 cs(\delta Z_{c_1} - \delta Z_{c_2})$$

$$\begin{aligned} c_A \cdots\to\cdots\times\cdots\leftarrow\cdots \bar{c}_Z &= ip^2 cs(\delta Z_{c_1} - \delta Z_{c_2}) - \\ &\quad - i\xi m_Z^2 cs\left[(\delta Z_{c_1} - \delta Z_{c_2}) + (\delta Z_{g_1} - \delta Z_{g_2})\right] \end{aligned}$$

The counterterms for pure scalar vertices are obtained as in U(2) theory by attaching appropriate factors to the corresponding Feynman rules.

$$\begin{aligned} \phi\phi\phi &: \text{tree} \times \left[\delta Z_\lambda + \frac{\delta v}{v}\right] \\ \phi\phi\phi\phi &: \text{tree} \times \delta Z_\lambda \end{aligned} \tag{A.12}$$

The counterterms for vertices involving gauge bosons become complicated due to the $A-Z$ mixing and different renormalization of their fields and couplings. There are also counterterms to vertices that do not appear at tree level. This is also the case for ghost vertices. For the vertices appearing already at tree level we use the same notations of momenta and indices below for their counterterms.

$G\phi\phi$ vertices:

$$\begin{aligned} A\sigma\sigma,\ A\pi\pi &: \text{tree} \times \left[\frac{(\delta Z_{g_1} + \delta Z_{g_2})}{2} + \delta Z_\phi\right] \\ Z\sigma\sigma,\ Z\pi\pi &: \text{tree} \times \left[\frac{(c^2\delta Z_{g_1} - s^2\delta Z_{g_2})}{(c^2 - s^2)} + \delta Z_\phi\right] \end{aligned}$$

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$$\begin{aligned}
Z\pi\sigma &: \text{tree} \times [(c^2\delta Z_{g_1} + s^2\delta Z_{g_2}) + \delta Z_\phi] \\
A_\mu\sigma(p_1)\pi(p_2) &: gcs(p_1-p_2)_\mu\ c_{12}(\delta Z_{g_1} - \delta Z_{g_2})
\end{aligned} \tag{A.13}$$

$GG\phi$ vertices:

$$\begin{aligned}
ZZ\sigma &: \text{tree} \times \left[2(c^2\delta Z_{g_1} + s^2\delta Z_{g_2}) + \delta Z_\phi + \frac{\delta v}{v}\right] \\
ZA\pi &: \text{tree} \times \left[(\delta Z_{g_1} + \delta Z_{g_2}) + \delta Z_\phi + \frac{\delta v}{v}\right] \\
Z_\mu(p_1)A_\nu(p_2)\sigma &: i2gcsm_Z g_{\mu\nu}\ c_{12}(\delta Z_{g_1} - \delta Z_{g_2})
\end{aligned} \tag{A.14}$$

$GG\phi\phi$ vertices:

$$\begin{aligned}
ZZ\sigma\sigma,\ ZZ\pi\pi &: i2g^2 g_{\mu\nu}\Big\{[2(c^4\delta Z_{g_1} + s^4\delta Z_{g_2}) + \left(c^4+s^4\right)\delta Z_\phi]c_{12}c_{34} + \\
&\qquad + 2c^2s^2\left[\delta Z_{g_1} + \delta Z_{g_2} + \delta Z_\phi\right] c_{13,24}\Big\} \\
ZZ\pi\sigma &: \text{tree} \times \left[\frac{2(c^4\delta Z_{g_1} - s^4\delta Z_{g_2})}{(c^2-s^2)} + \delta Z_\phi\right] \\
AA\sigma\sigma,\ AA\pi\pi &: \text{tree} \times [\delta Z_{g_1} + \delta Z_{g_2} + \delta Z_\phi] \\
AZ\sigma\sigma,\ AZ\pi\pi &: i2g^2 csg_{\mu\nu}\Big\{[2(c^2\delta Z_{g_1} - s^2\delta Z_{g_2}) + \left(c^2-s^2\right)\delta Z_\phi]c_{12}c_{34} - \\
&\qquad - \left(c^2-s^2\right)\left[\delta Z_{g_1} + \delta Z_{g_2} + \delta Z_\phi\right] c_{13,24}\Big\} \\
AZ\pi\sigma &: i2g^2 csg_{\mu\nu}\Big\{[2(c^2\delta Z_{g_1} + s^2\delta Z_{g_2}) + \delta Z_\phi]c_{12}s_{34} + \\
&\qquad + \left[\delta Z_{g_1} + \delta Z_{g_2} + \delta Z_\phi\right] s_{13,24}\Big\} \\
A_\mu(p_1)A_\nu(p_2)\pi(p_3)\sigma(p_4) &: i4g^2c^2s^2 g_{\mu\nu}\ c_{12}s_{34}(\delta Z_{g_1} - \delta Z_{g_2})
\end{aligned} \tag{A.15}$$

GGG vertices:

$$\begin{aligned}
AAA &: \text{tree} \times [(s^2\delta Z_{g_1} + c^2\delta Z_{g_2}) + (s^2\delta Z_{G_1} + c^2\delta Z_{G_2})] \\
ZZZ &: \text{tree} \times \frac{(c^4\delta Z_{g_1} - s^4\delta Z_{g_2}) + (c^4\delta Z_{G_1} - s^4\delta Z_{G_2})}{(c^2-s^2)} \\
ZZA &: \text{tree} \times [(c^2\delta Z_{g_1} + s^2\delta Z_{g_2}) + (c^2\delta Z_{G_1} + s^2\delta Z_{G_2})] \\
A_\alpha(p_1)A_\beta(p_2)Z_\gamma(p_3) &: -2gc^2s^2\ s_{12}\ P_{\alpha\beta\gamma}(p_1,p_2,p_3)[(\delta Z_{g_1} - \delta Z_{g_2}) + (\delta Z_{G_1} - \delta Z_{G_2})]
\end{aligned} \tag{A.16}$$

$GGGG$ vertices:

$$\begin{aligned}
AAAA &: \text{tree} \times [2(s^2\delta Z_{g_1} + c^2\delta Z_{g_2}) + (s^2\delta Z_{G_1} + c^2\delta Z_{G_2})] \\
ZZZZ &: \text{tree} \times \frac{1}{(c^6+s^6)} \times \\
&\qquad \times[2(c^6\delta Z_{g_1} + s^6\delta Z_{g_2}) + (c^6\delta Z_{G_1} + s^6\delta Z_{G_2})] \\
ZZAA &: \text{tree} \times [2(c^2\delta Z_{g_1} + s^2\delta Z_{g_2}) + (c^2\delta Z_{G_1} + s^2\delta Z_{G_2})] \\
ZZZA &: \text{tree} \times \frac{1}{(c^2-s^2)} \times \\
&\qquad \times[2(c^4\delta Z_{g_1} - s^4\delta Z_{g_2}) + (c^4\delta Z_{G_1} - s^4\delta Z_{G_2})]
\end{aligned}$$

$$
\begin{aligned}
A_{\mu_1}(p_1)A_{\mu_2}(p_2)A_{\mu_3}(p_3)Z_{\mu_4}(p_4) \, &: i4g^2c^3s^3[2(\delta Z_{g_1} - \delta Z_{g_2}) + (\delta Z_{G_1} - \delta Z_{G_2})] \times \\
&\quad \times [g_{\mu_4\mu_1}g_{\mu_2\mu_3}(c_{43,12} - c_{41}c_{23}) + \\
&\qquad + g_{\mu_4\mu_2}g_{\mu_3\mu_1}(c_{41,23} - c_{42}c_{31}) + \\
&\qquad + g_{\mu_4\mu_3}g_{\mu_1\mu_2}(c_{42,31} - c_{43}c_{12})]
\end{aligned}
\tag{A.17}
$$

$\phi c\bar{c}$ vertices:

$$
\begin{aligned}
\sigma c_Z \bar{c}_Z \, &: \text{tree} \times [(c^2\delta Z_{g_1} + s^2\delta Z_{g_2}) + (c^2\delta Z_{c_1} + s^2\delta Z_{c_2})] \\
\pi c_Z \bar{c}_Z \, &: \text{tree} \times \frac{(c^2\delta Z_{g_1} - s^2\delta Z_{g_2}) + (c^2\delta Z_{c_1} - s^2\delta Z_{c_2})}{(c^2 - s^2)} \\
\pi c_A \bar{c}_Z \, &: \text{tree} \times \frac{(\delta Z_{g_1} + \delta Z_{g_2}) + (\delta Z_{c_1} + \delta Z_{c_2})}{2} \\
\sigma c_A(p_1)\bar{c}_Z(p_2) \, &: -i\xi g^2 vcs \; c_{21}[(\delta Z_{g_1} - \delta Z_{g_2}) + (\delta Z_{c_1} - \delta Z_{c_2})]
\end{aligned}
\tag{A.18}
$$

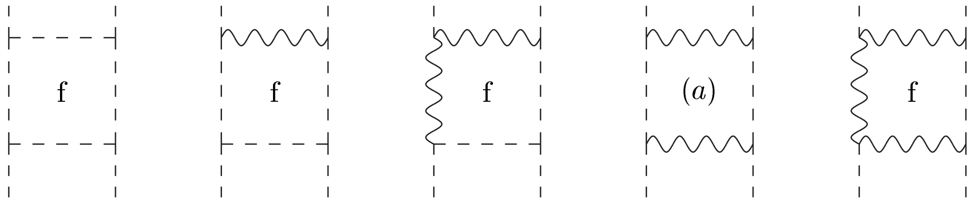
$Gc\bar{c}$ vertices:

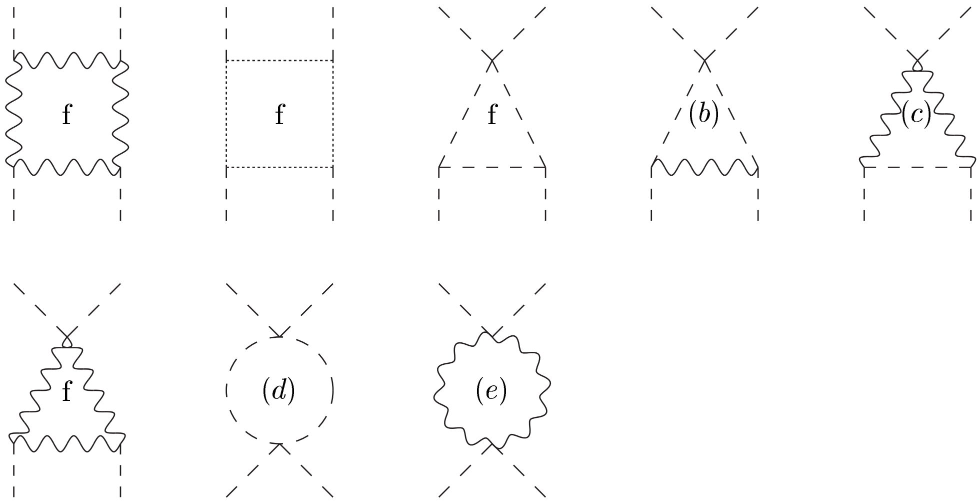
$$
\begin{aligned}
A c_A \bar{c}_A \, &: \text{tree} \times [(s^2\delta Z_{g_1} + c^2\delta Z_{g_2}) + (s^2\delta Z_{c_1} + c^2\delta Z_{c_2})] \\
A c_Z \bar{c}_Z \, &: \text{tree} \times [(c^2\delta Z_{g_1} + s^2\delta Z_{g_2}) + (c^2\delta Z_{c_1} + s^2\delta Z_{c_2})] \\
Z c_Z \bar{c}_Z \, &: \text{tree} \times \frac{1}{(c^2 - s^2)} \times \\
&\quad \times [(c^4\delta Z_{g_1} - s^4\delta Z_{g_2}) + (c^4\delta Z_{c_1} - s^4\delta Z_{c_2})] \\
Z c_A \bar{c}_Z, \; Z c_Z \bar{c}_A \, &: \text{tree} \times [(c^2\delta Z_{g_1} + s^2\delta Z_{g_2}) + (c^2\delta Z_{c_1} + s^2\delta Z_{c_2})] \\
A_\mu c_A(p_1)\bar{c}_Z(p_2), \; A_\mu c_Z(p_1)\bar{c}_A(p_2) \, &: 2gc^2s^2 p_{2\mu} \; s_{21}[(\delta Z_{g_1} - \delta Z_{g_2}) + (\delta Z_{c_1} - \delta Z_{c_2})] \\
Z_\mu c_A(p_1)\bar{c}_A(p_2) \, &: 2gc^2s^2 p_{2\mu} \; s_{21}[(\delta Z_{g_1} - \delta Z_{g_2}) + (\delta Z_{c_1} - \delta Z_{c_2})]
\end{aligned}
\tag{A.19}
$$

B. One loop diagrams for 1PI four-point functions

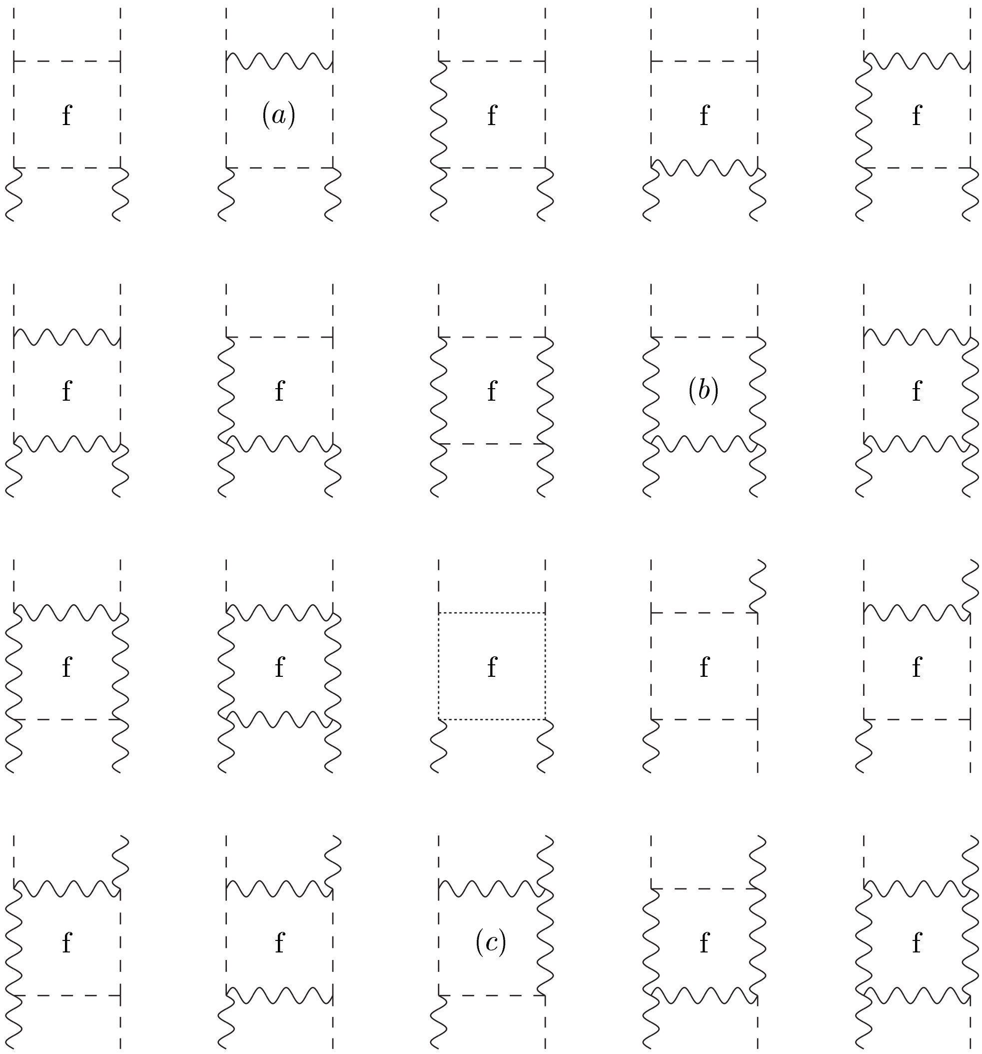
We show below topologically different diagrams in which the wavy, dashed and dotted lines represent the gauge, scalar and ghost fields respectively. For a concrete vertex all possible assignments of fields must be included. Diagrams with an " f " are finite by power counting. The diagrams for two- and three-point functions are similar to those in U(2) theory which are shown in ref. [10], with the exclusion of the charged particles, and will not be repeated here.

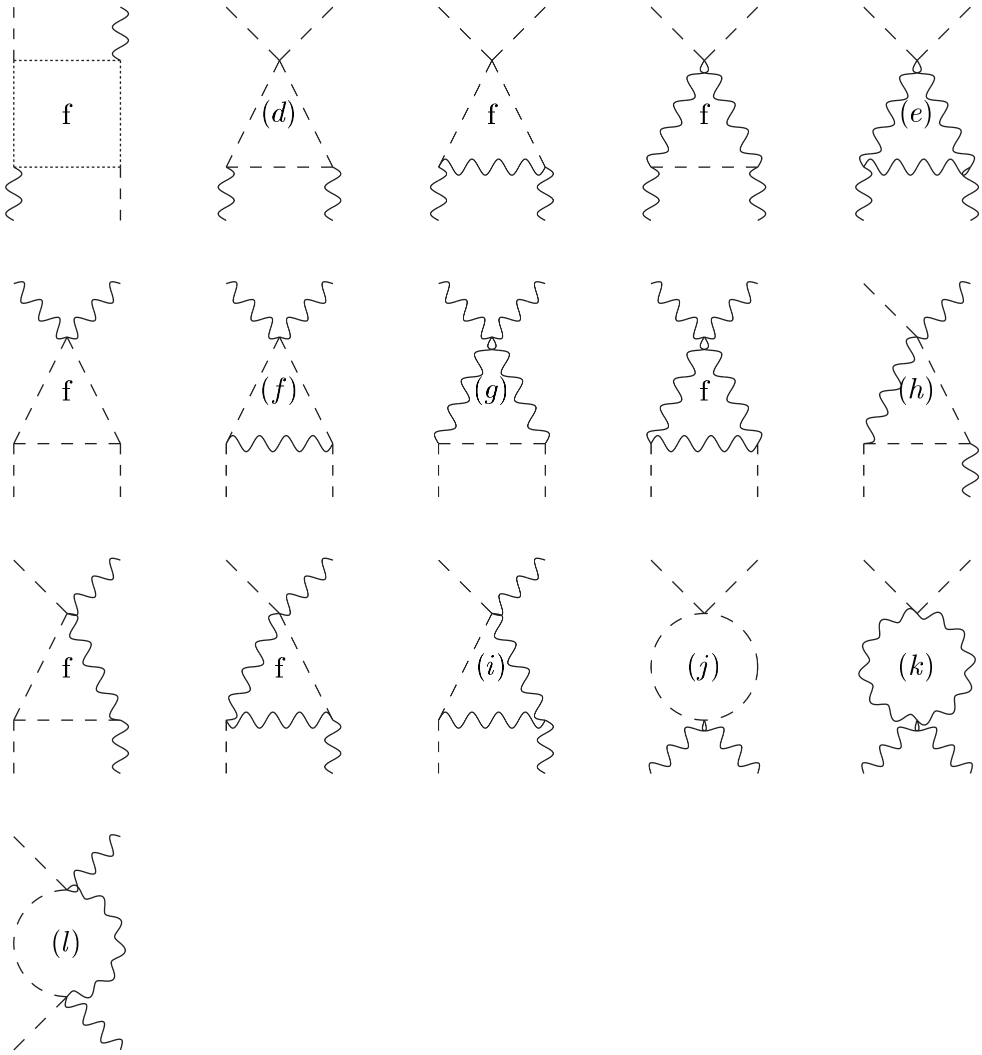
$\phi\phi\phi\phi$ vertex:



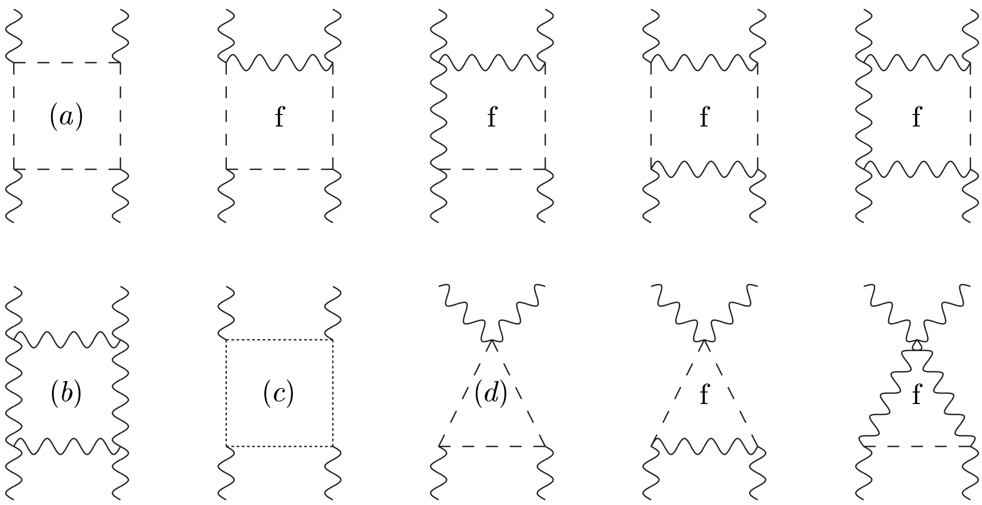


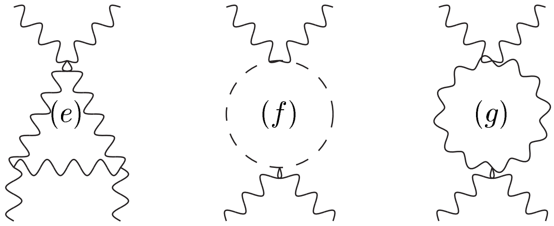
$\phi\phi GG$ vertex:





GGGG vertex:





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